Statistical Distributions in R

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While writing the [Randomness Concept][concept-randomness] for the Exercism learning syllabus, there was some uncertainty about what all the various R functions do. Here, we demonstrate several them in action.

The `q*()` functions, to generate the quantiles, are not (yet) included.

First, we need the Tidyverse libraries and some default settings:

```r
library(tidyverse)

set_theme(theme_gray(base_size = 16))
```

1 Normal/Gaussian Distribution

This is the classic “bell-shaped curve”.

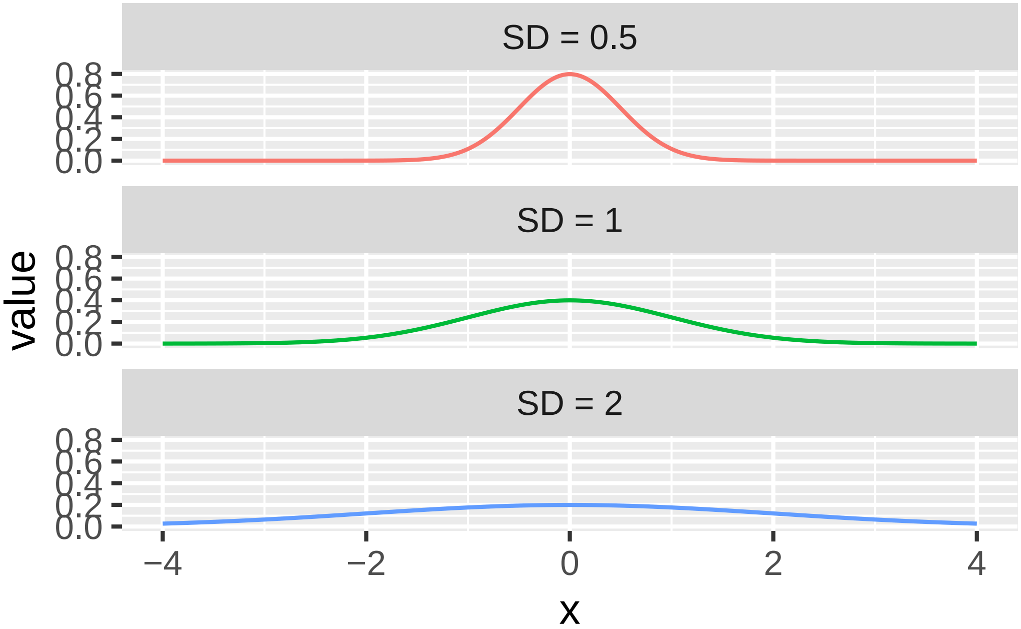
1.1 The Normal Probability Density Function

The `dnorm()` function calculates the PDF at a point: the height of the curve at that point.

Different values for standard deviation are shown. Higher values lead to wider, flatter curves.

All means default to zero.

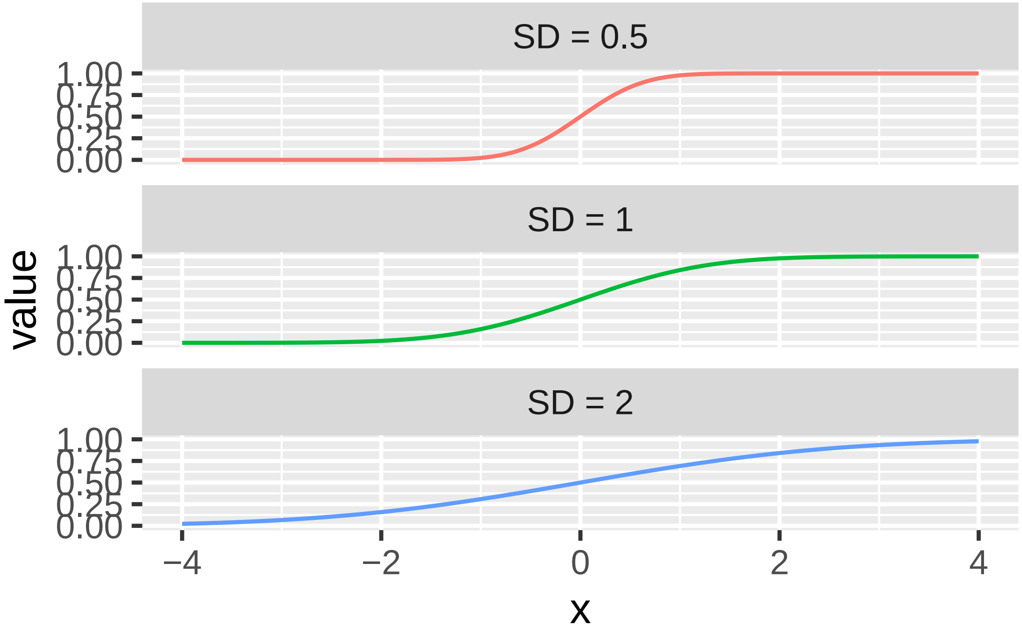
```r
tibble(
  "x" = seq(-4, 4, length.out = 1000),
  "SD = 0.5" = dnorm(x, sd = 0.5),
  "SD = 1" = dnorm(x, sd = 1),
  "SD = 2" = dnorm(x, sd = 2)
)  |>
  pivot_longer(
    cols = starts_with("SD = "),
    names_to = "SD",
    values_to = "value"
  ) |>
  ggplot(aes(x = x, y = value, color = SD)) +
    geom_line() +
    facet_wrap(~SD, ncol = 1) +
    theme(legend.position = "none")
```



1.2 The Normal Cumulative Distribution Function

The `pnorm()` function calculates the CDF at a point: the integral of the curve to the left of that point.

```r
tibble(
  "x" = seq(-4, 4, length.out = 1000),
  "SD = 0.5" = pnorm(x, sd = 0.5),
  "SD = 1" = pnorm(x, sd = 1),
  "SD = 2" = pnorm(x, sd = 2)
)  |>
  pivot_longer(
    cols = starts_with("SD = "),
    names_to = "SD",
    values_to = "value"
  ) |>
  ggplot(aes(x = x, y = value, color = SD)) +
    geom_line() +
    facet_wrap(~SD, ncol = 1) +
    theme(legend.position = "none")
```

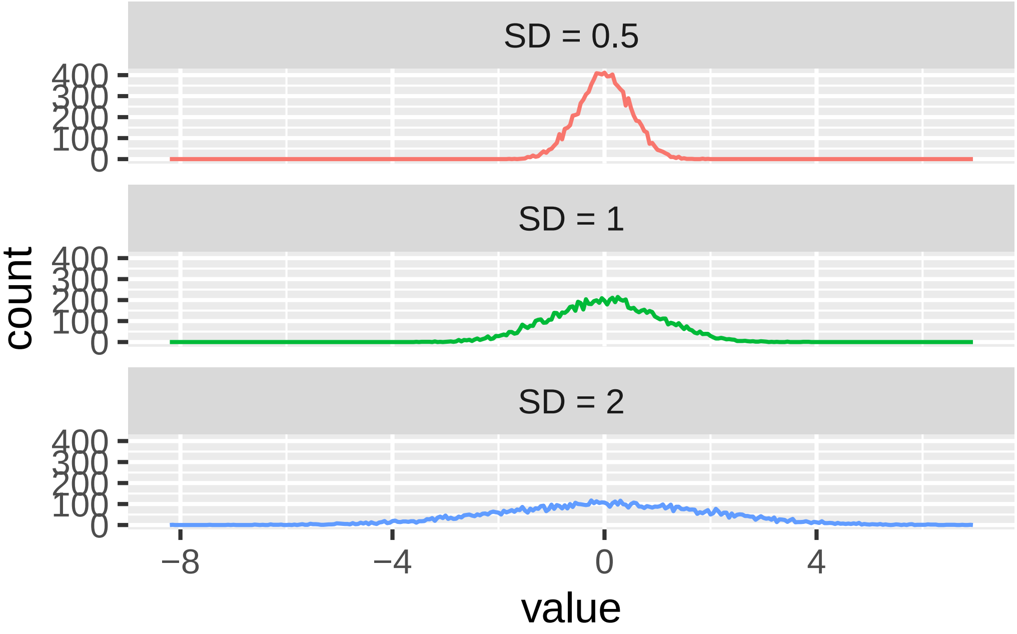


1.3 Generating random values with `rnorm()`

The `rnorm()` function generates random values which are normally distributed.

Here, we get 10,000 values, using a frequency polygon to bin them, and count number in each bin for the `y` axis.

```r
n <- 10000
tibble(
  "SD = 0.5" = rnorm(n, sd = 0.5),
  "SD = 1" = rnorm(n, sd = 1),
  "SD = 2" = rnorm(n, sd = 2)
)  |>
  pivot_longer(
    cols = starts_with("SD = "),
    names_to = "SD",
    values_to = "value"
  ) |>
  ggplot(aes(value, color = SD)) +
    geom_freqpoly(binwidth = 0.05) +
    facet_wrap(~SD, ncol = 1) +
    theme(legend.position = "none")
```



2 Binomial Distribution

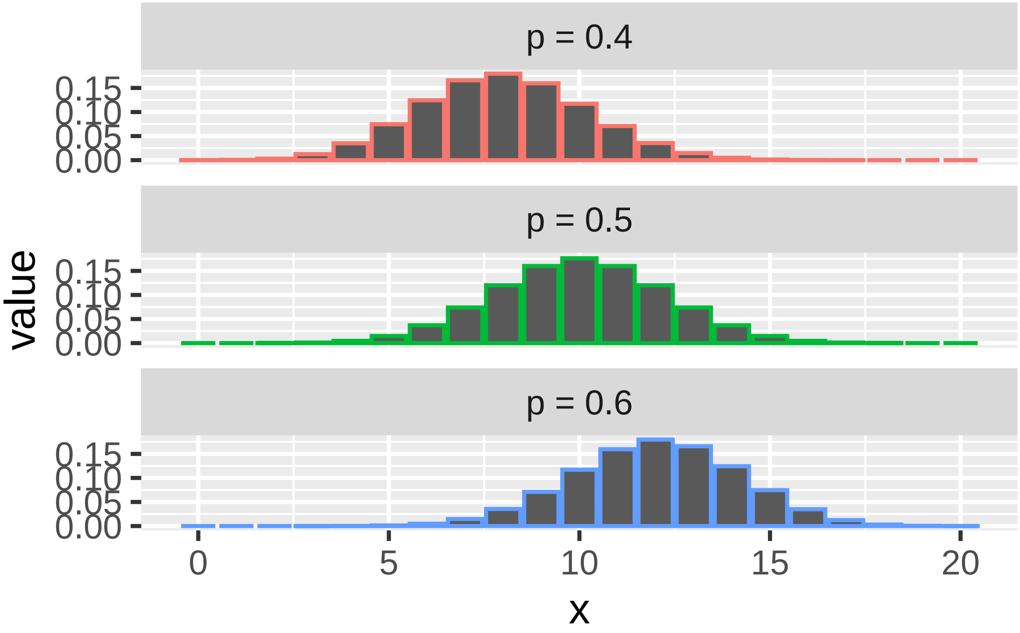
Here we use the example of 20 coin flips, counting the number that come up heads.

Three probabilities are used:

- `p = 0.4` for a coin biased to tails.
- `p = 0.5` for a fair coin.
- `p = 0.6` for a coin biased to heads.

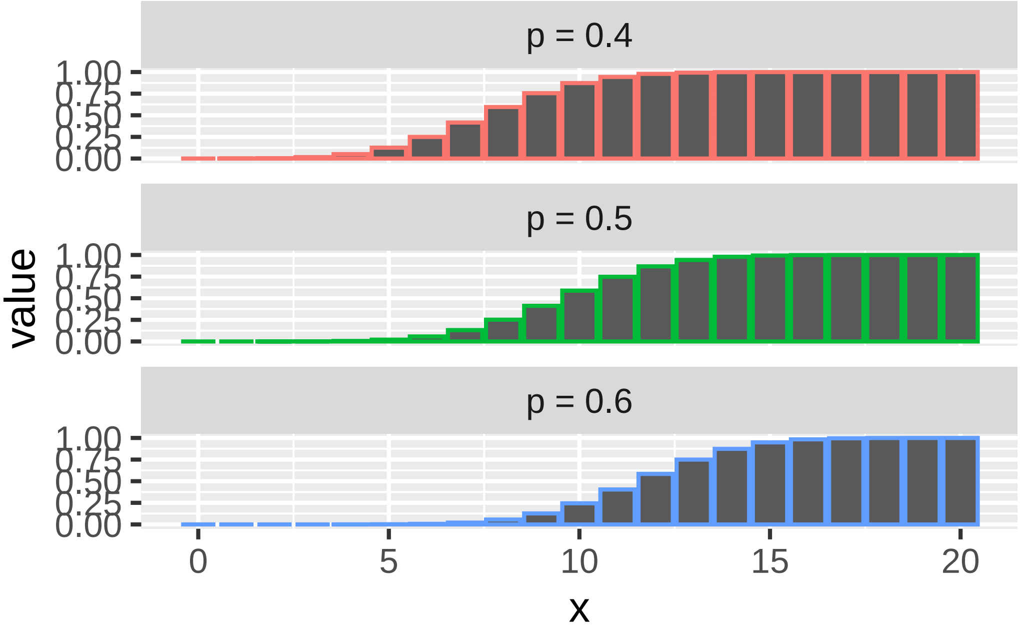
2.1 The Binomial Probability Density Function

```
tibble(
  "x" = 0:20,
  "p = 0.4" = dbinom(x, size = 20, prob = 0.4),
  "p = 0.5" = dbinom(x, size = 20, prob = 0.5),
  "p = 0.6" = dbinom(x, size = 20, prob = 0.6)
)  |>
  pivot_longer(
    cols = starts_with("p = "),
    names_to = "p",
    values_to = "value"
  ) |>
  ggplot(aes(x = x, y = value, color = p)) +
    geom_col() +
    facet_wrap(~p, ncol = 1) +
    theme(legend.position = "none")
```



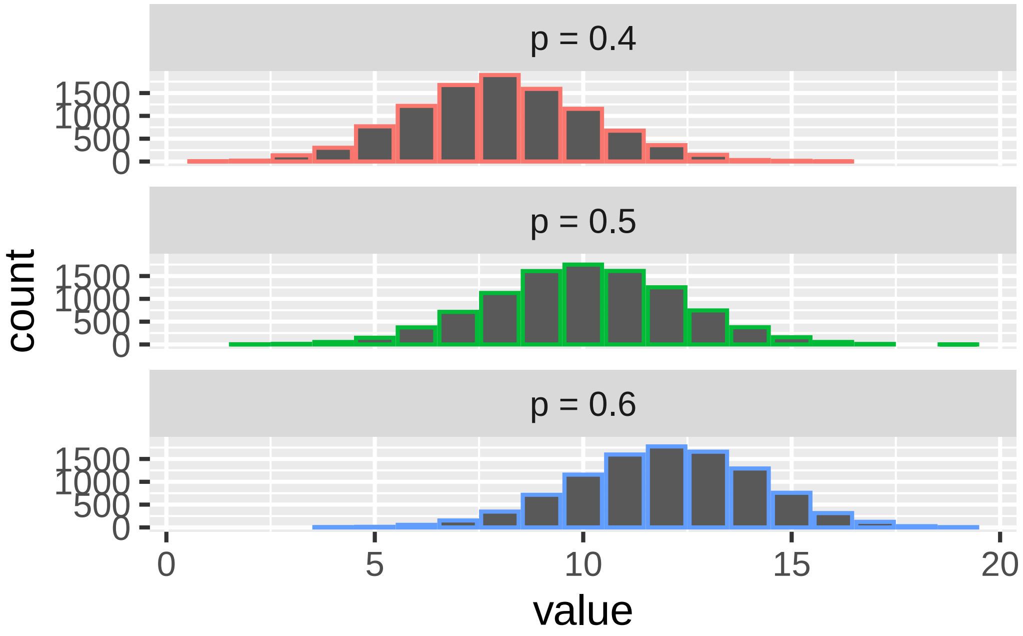
2.2 The Binomial Cumulative Distribution Function

```
tibble(
  "x" = 0:20,
  "p = 0.4" = pbinom(x, size = 20, prob = 0.4),
  "p = 0.5" = pbinom(x, size = 20, prob = 0.5),
  "p = 0.6" = pbinom(x, size = 20, prob = 0.6)
)  |>
  pivot_longer(
    cols = starts_with("p = "),
    names_to = "p",
    values_to = "value"
  ) |>
  ggplot(aes(x = x, y = value, color = p)) +
  geom_col() +
  facet_wrap(~p, ncol = 1) +
  theme(legend.position = "none")
```



2.3 Generating random values with `rbinom()`

```r
n <- 10000
tibble(
  "p = 0.4" = rbinom(n, size = 20, prob = 0.4),
  "p = 0.5" = rbinom(n, size = 20, prob = 0.5),
  "p = 0.6" = rbinom(n, size = 20, prob = 0.6)
)  |>
  pivot_longer(
    cols = starts_with("p = "),
    names_to = "p",
    values_to = "value"
  ) |>
  ggplot(aes(value, color = p)) +
  geom_bar() +
  facet_wrap(~p, ncol = 1) +
  theme(legend.position = "none")
```



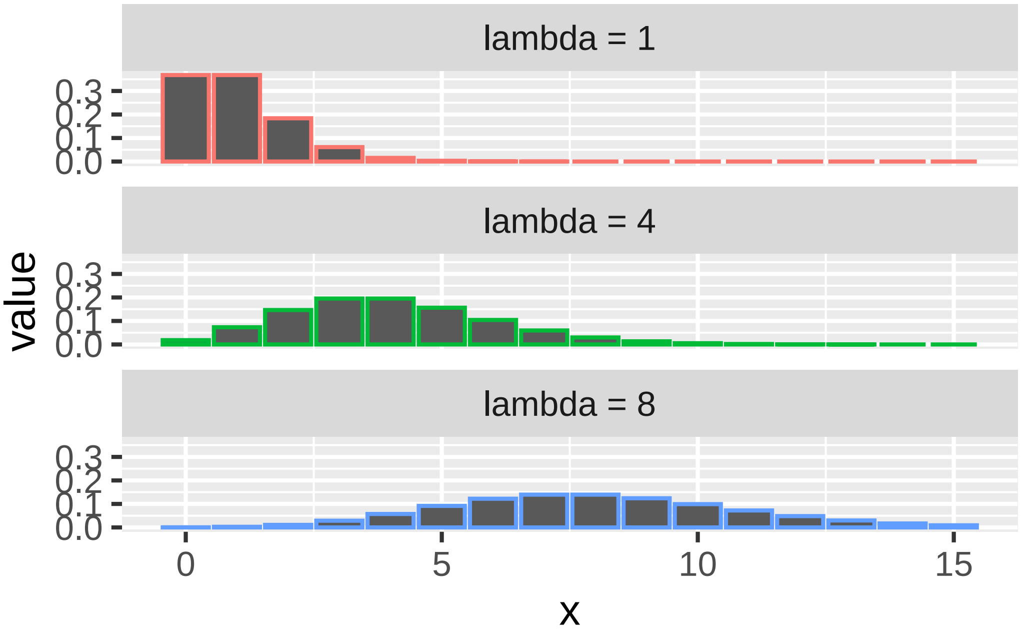
3 Poisson Distribution

Here we use the example of how many meteorites at least 1m in diameter strike the Earth each year.

The `lambda` values are different assumptions for the *average* number. The [Catalina Sky Survey][web-catalina] discuss this and estimate such strikes happen “every few months”, so `lambda = 4` is a reasonable guess.

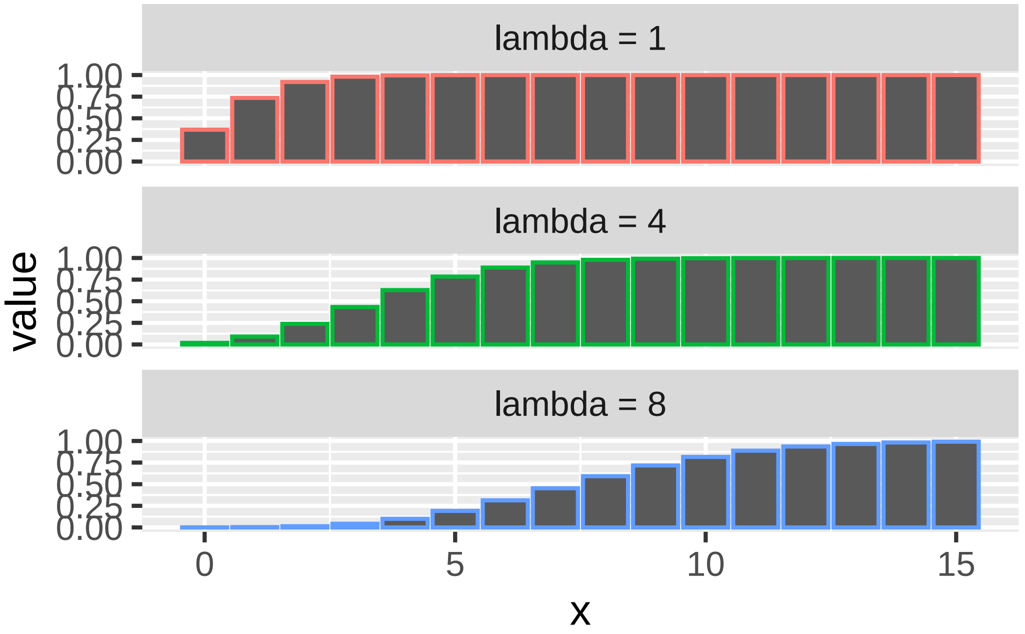
3.1 The Poisson Probability Density Function

```
tibble(
  "x" = 0:15,
  "lambda = 1" = dpois(x, lambda = 1),
  "lambda = 4" = dpois(x, lambda = 4),
  "lambda = 8" = dpois(x, lambda = 8)
)  |>
  pivot_longer(
    cols = starts_with("lambda = "),
    names_to = "lambda",
    values_to = "value"
  ) |>
  ggplot(aes(x = x, y = value, color = lambda)) +
    geom_col() +
    facet_wrap(~lambda, ncol = 1) +
    theme(legend.position = "none")
```



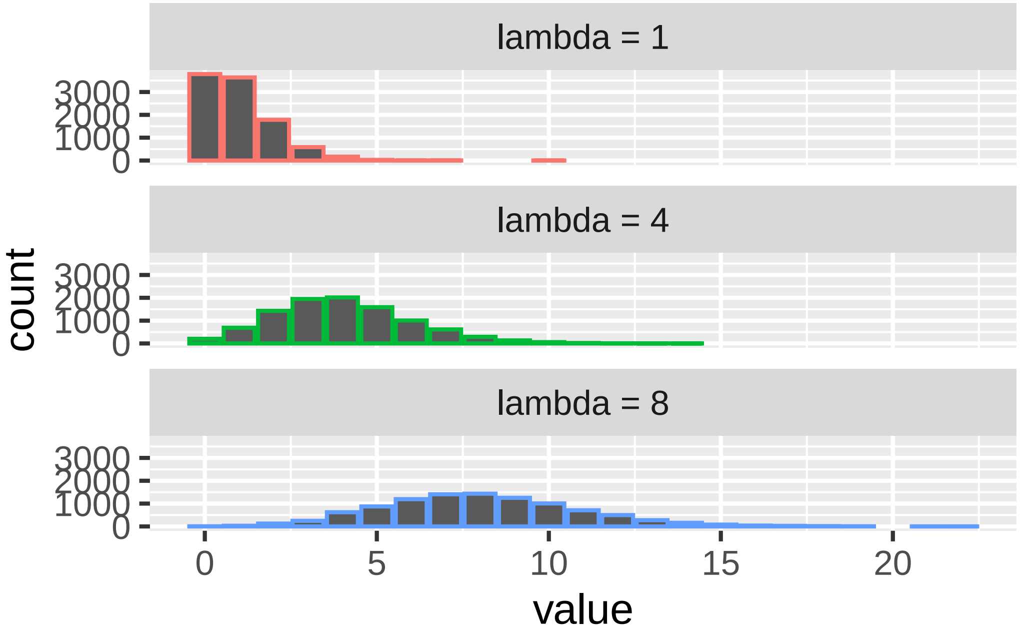
3.2 The Poisson Cumulative Distribution Function

```r
tibble(
  "x" = 0:15,
  "lambda = 1" = ppois(x, lambda = 1),
  "lambda = 4" = ppois(x, lambda = 4),
  "lambda = 8" = ppois(x, lambda = 8)
)  |>
  pivot_longer(
    cols = starts_with("lambda = "),
    names_to = "lambda",
    values_to = "value"
  ) |>
  ggplot(aes(x = x, y = value, color = lambda)) +
    geom_col() +
    facet_wrap(~lambda, ncol = 1) +
    theme(legend.position = "none")
```



3.3 Generating random values with `rpois()`

```
n <- 10000
tibble(
  "lambda = 1" = rpois(n, lambda = 1),
  "lambda = 4" = rpois(n, lambda = 4),
  "lambda = 8" = rpois(n, lambda = 8)
)  |>
  pivot_longer(
    cols = starts_with("lambda = "),
    names_to = "lambda",
    values_to = "value"
  ) |>
  ggplot(aes(value, color = lambda)) +
    geom_bar() +
    facet_wrap(~lambda, ncol = 1) +
    theme(legend.position = "none")
```


lambda = 1
lambda = 4
lambda = 8
count
3000
2000
1000
0
0
5
10
15
20
value